Typos that occur in our encyclopedic monograph and additional material

Extension Problems and Stable Ranks—*A Space Odyssey*
Birkhäuser 2021, 2198 p.

4.9.2023

page	line			
3	-8	It said	to be replaced by	It is said
7	3	$\phi(t) \neq 0$	to be replaced by	$\varphi(t) \neq 0$
7	5	x, y, X	to be replaced by	$s, t, [0, \infty]$
13	2	just proved	to be deleted	
14	-7		to be added	If A and B are two subsets of X, then A is *dense with respect to* B if $\overline{A} \supseteq B$.
19	1	T_1 axiom	to be replaced by	T_1-axiom
29	$10, 11, 14, 15, 16$	n	to be replaced 6 times by	m
33	-17	F_σ subset	to be replaced by	F_σ-subset
34	10	Since, X	to be replaced by	Since X
34	14	T_1 space	to be replaced by	T_1-space
34	-3	if $X = \mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$	to be deleted	
38	-1	.	to be deleted	
39	7	$j = 1, \dots, N$	to be replaced by	$j = 1, \dots, n$
41	8	an homeomorphism	to be replaced by	a homeomorphism
42	12	$\tilde{x} := \{y : x \sim y\}$	to be enlarged by	$\tilde{x} := \{y : x \sim y\} = [x]$
43	5	$\tilde{f}(\tilde{x}) = \tilde{f}(\tilde{y})$	to be replaced by	$\tilde{f}([x]) = \tilde{f}([y])$
51	-1	S^1	to be replaced by	S_1
52	$4, 5, 7, 10$	S^1	to be replaced by	S_1
61	16	x_0; which	to be replaced by	x_0, which
61	19	subset set	to be replaced by	subset
61	-1	to x_2	to be replaced by	to x_1
66	20	$1/N' - 1$	to be replaced by	$1/(N' - 1)$
96	5	hypotheses	to be replaced by	hypothesis

page	line			
104	-6	immmediately	to be replaced by	immediately
105	6	injective.	to be replaced by	injective:
109	13	of	to be replaced by	or
111	13	$(M\times S)\cup(S\times M)\cup(S\times S)$	to be replaced by	$(M\times M)\cup(M\times S)\cup(S\times M)\cup(S\times S)$
129	-10	$\mathcal{R}\setminus\mathcal{F}$ The	period missing	$\mathcal{R}\setminus\mathcal{F}$. The
150	11	Theorem 1.310	to be replaced by	Proposition 1.310
161	7	Theorem 1.316	to be replaced by	Corollary 1.316
197	-5	square in $\mathbb{C}$	to be replaced by	square in $\mathbb{R}^2$
200	-1	$[a,b]^n, a<b,$	to be replaced by	$\prod_{j=1}^n [c_j, c_j+r],\ c_j\in\mathbb{R},\ r>0,$
207	3	Lemma 3.42	to be replaced by	Corollary 3.42
210	18	Lemma 3.48	to be replaced by	Proposition 3.48
217	-8	$\Phi(1),\Phi'(1)$	to be replaced by	$\Phi(2\pi),\Phi'(2\pi)$
217	-7	$\phi(e^{it})$	to be replaced by	$\phi(e^{it}) := \Phi(t)$
229	-3	Γ	to be replaced by	γ
236	-7	$G\in C^k(\mathbb{C})$	to be replaced by	$G\in C^n(\mathbb{C})$
243	-8	$\leq 1/e$	to be replaced by	<1
243	$-5,-6$	$1/e$	to be replaced by	1
243	-5	K°	to be replaced by	$\mathbb{D}$
246	8	Lebesgue's majoration theorem	to be replaced by	Lebesgue's dominated convergence theorem
248	-1			period . is missing
254	-7	$\gamma^* =$	to be replaced by	$\gamma^* :=$
259	-5	$U\subseteq\mathbb{C}$	to be replaced by	$U\subseteq\mathbb{R}^2$
269	$-3,-4,-5$	$d\zeta$	to be replaced three times by	$d\sigma_2(\zeta)$
276	3	$\mathbb{C}$	to be replaced by	$\widehat{\mathbb{C}}$
276	8	Lebesgue measure	to be replaced by	planar Lebesgue measure
282	3	$-\int_{\lvert\xi\rvert=1}\frac{\overline{q(\xi)}}{a\xi}$	to be replaced by	$-\int_{\lvert\xi\rvert=1}\frac{\overline{q(\xi)}}{a\xi}\,d\xi$

page	line			
310	-7	$p_{n+1}(t) =$	to be replaced by	$p_{n+1}(t) :=$
312	12	is dense	to be replaced by	is uniformly dense
313	7	dense.	to be replaced by	dense in $C(X, \mathbb{R})$.
315	3	K	to be replaced by	$\mathbb{K}$
316	-10	$+\ell_n$	to be replaced by	$-\ell_n$
354	15	$\mathrm{Res}\,(f, w)$	to be replaced by	$\mathrm{Res}\,(\frac{f'}{f}, w)$
363	-2	(3)	to be replaced by	(4)
381	-3	K	to be replaced by	$\mathbb{K}$
392	2	Riesz	to be replaced by	F. Riesz
396	2	$\mathbb{R}^+$	to be replaced by	$[0, \infty[$
415	-4		to be added at the beginning of the line	$(x, y) \mapsto xy$
420	-6	Corollary	to be replaced by	Proposition
422	5	Cauchy-Schwarz	to be replaced by	the Cauchy-Schwarz inequality
422	-4	(f_n)	to be replaced by	(f_k)
422	-1	n	to be replaced tree times by	k

page	line			
423	4	n	to be replaced tree times by	k
431	−14	$\frac{1}{1-\|\|\mathbf{1}-f\|\|}$	to be replaced by	$\frac{\|\|\mathbf{1}\|\|}{1-\|\|\mathbf{1}-f\|\|}$
432	2		to be added	and we may assume that $\|\|\mathbf{1}\|\| \geq 1$.
433	5	*resolvent* set	to be replaced by	*resolvent set*
434	−1	$\|\|a^n\|\|^{1/n}$	to be replaced by	$\|\|a_n\|\|^{1/n}$
445	$4-8$	1	to be replaced 6 times by	$\mathbf{1}$
459	−2	no	to be replaced by	not a
462	2	Proposition	to be replaced by	Example
464	7	Theorem	to be replaced by	Corollary
467	4	z	to be replaced by	Z
474	−7	Theorem	to be replaced by	Corollary
474	−10	in in Theorem	to be replaced by	in Corollary
480	3	(3)	to be replaced by	(iii)
480	12	Theorem,	to be replaced by	Theorem
486	−1	f_j	to be replaced by	$\widehat{f_j}$
497	11	z	to be replaced by	$(z_1, \ldots, z_n)$

page	line			
500	-5	K_n	to be replaced by	K_N
503	18		to be added	For the sake of simplicity, we identify $\alpha \cdot \mathbf{1} \in A$ with the scalar α
504	9		delete	(see also Theorem 33.17)
520	7	Proposition	to be replaced by	Corollary
522	-6	$g - \widehat{g}(x)$	to be replaced by	$g - \widehat{g}(x) \cdot \mathbf{1}$
534	7	as soon as	to be replaced by	provided
536	-3	$\kappa([q)]$	to be replaced by	$\kappa([q])$
537	-2	Theorem	to be replaced by	Proposition
537	-1	invariance	to be deleted	
550	2	Then	to be replaced by	Moreover,
557	18	$\|\|1 - f^{-1}g\|\|$	to be replaced by	$\|\|\mathbf{1} - f^{-1}g\|\|$
561	-7	Lemma	to be replaced by	Proposition
564	7	M_n	to be replaced by	M_j
565	-13	Proposition	to be replaced by	Appendix
565	-13	in the appendix	to be deleted	
566	6	curve $\Phi : \exp A \to$	to be replaced by	path $\Phi : [0,1] \to$

page	line			
566	9	$e^{\text{trace } X}$	to be replaced by	$e^{\text{tr } X}$
567	5	Theorem	to be replaced by	Appendix
565	−13	in the appendix	to be deleted	
568	−2	and	to be replaced by	and so
574	5	Proposition	to be replaced by	Theorem
575	10	Proposition	to be replaced by	Theorem
575	−7	$X(A)$	to be replaced by	$M(A)$
576	−1	$X(A)$	to be replaced by	$M(A)$
576	5	Proposition	to be replaced by	Theorem
583	−11	$2\ell\pi i$	to be replaced twice by	$2\ell\pi i \cdot \mathbf{1}$
584	11	$2\ell\pi i$	to be replaced twice by	$2\ell\pi i \cdot \mathbf{1}$
588	9	$e^{-g/2+(j-m)\pi i}$	to be replaced by	$e^{-g/2+(j-m)\pi i \cdot \mathbf{1}}$
589	4	$2j\pi i$	to be replaced twice by	$2j\pi i \cdot \mathbf{1}$
589	−3	$2j\pi i$	to be replaced by	$2j\pi i \cdot \mathbf{1}$
589	footnote	$2k\pi i$	to be replaced by	$2k\pi i \cdot \mathbf{1}$
597	−1	and note that $\Phi(\mathbf{1}_A) = \mathbf{1}_B$	to be added	

page	line			
600	8	is matrix	to be replaced by	is a matrix
625	−10	$w := r + sZ \in \mathcal{R}$ and that	to be replaced by	the function w given by $w(z) = r + sz$ belongs to $\mathcal{R}$ and satisfies
629	−4	yy'	to be replaced by	$y'y$
630	16	$\lvert\lambda\rvert^2$	to be replaced by	$\lvert\lambda\rvert^2 \cdot \mathbf{1}$
630	$16, 17, -8$	β^2	to be replaced by	$\beta^2 \cdot \mathbf{1}$
633	10	σ	to be replaced by	$\sigma \neq 0$
633	−5	every	to be replaced by	this
635	11	$\sigma^*_{\mathcal{R}}(f\langle a\rangle)$	to be replaced by	$\sigma^*_{\mathcal{R}}(f[a])$
653	2	in	to be deleted	
655	9	1	to be replaced twice by	$\mathbf{1}$
655	13	R	to be replaced by	$\mathcal{R}$
655	−14	1	to be replaced by	$\mathbf{1}$
656	−16	the the	to be replaced by	the
656	−10	Corollary	to be replaced by	Theorem
668	3	:	to be replaced by	, where $\mathcal{R} \in \mathscr{R}$:
691	1	(SC2)	to be replaced by	(SC2))

page	line			
691	2	$\left\lvert \sqrt[n]{\lvert 1-z\rvert}\right.$	to be replaced by	$\left\lvert (\sqrt[n]{\lvert 1-z\rvert}\right.$
721	4	$\left\lVert \frac{z-1}{z-r_n}\, p \right\rVert$	to be replaced by	$\left\lVert \frac{z-1}{z-r_n}\, p \right\rVert_{W^+}$
725	7	have approximate	to be replaced by	have bounded approximate
738	12	Max	to be replaced by	Max
742	-6	$M+\mathbb{K}\,a+A\,a$	to be replaced by	$M+\mathbb{C}\,a+A\,a$
744	7	/commutative	to be deleted	
744	8		add as a foonote	Of course this operation can also be defined on unital Banach algebras
744	-1	$\lvert\lvert(1-a)^{-1}x\rvert\rvert$	to be replaced by	$\lvert\lvert(1-a)^{-1}x\rvert\rvert_u$
750	5	necessary	to be replaced by	necessarily
751	-15	$M(B)$	to be replaced by	$M_{\mathbb{K}}(B)$
751	-4	linear	to be replaced by	$\mathbb{K}$-linear
752	4	$\mathbb{C}$	to be replaced by	$\mathbb{K}$
757	17	over $\mathbb{K}$	to be replaced by	over $\mathbb{C}$
760	-16	Theorem	to be replaced by	Proposition
760	-2	$\mathbb{C}$	to be replaced by	$\mathbb{K}$
761	4	Proposition	to be replaced by	Lemma
761	$12, -3$	$\mathbb{C}$	to be replaced by	$\mathbb{K}$

page	line			
761	16		line to be replaced by	Also equivalent are:
762	5	$\mathbb{C}$	to be replaced by	$\mathbb{K}$
762	-5	Proposition	to be replaced by	Propositions
763	$3, 7$	$\mathbb{C}$	to be replaced twice by	$\mathbb{K}$
763	-9	field	to be replaced by	division algebra
763	$-4, -7$	1	to be replaced twice by	$\mathbf{1}$
763	-3	$\mathbb{C}$	to be replaced twice by	$\mathbb{K}$
763	footnote 239	coincides with	to be replaced by	is similar to
764	$5, 10$	1	to be replaced twice by	$\mathbf{1}$
768	12	$*1$	to be replaced by	$*\mathbb{E}$
768	-12	$\varepsilon.$	to be replaced by	ε
769	$-3, -5$	1^*	to be replaced three times by	$\mathbb{E}^*$
784	-1	(and are true in view of Theorem 10.2):	to be deleted	actually wrong for $\dim E = \infty$
785	-3	Theorem 10.12	add	combined with Theorem 10.2
796	$-10, -11$	$\|f_j^*\|$	to be replaced by	$\|f_j^*\|^2$
803	18	zero-free	to be replaced by	zero-free function
803	22	$e^{tL(z)}$	to be replaced by	$e^{(1-t)L(z)}$

page	line			
804	-15	can be taken to be	to be replaced by	is
804	-10	$e^{\psi} = \phi$	to be replaced by	$e^{\psi} = \varphi$
804	-1	$e^{\psi_n} = \varphi\vert_{[n,n+1]}$	to be replaced by	$e^{\psi_n} = \phi\vert_{[n,n+1]}$
805	5	$\exp\left(\frac{1}{2}L_n(\tau(t))\right)$	to be replaced by	$\exp\left(\frac{1}{2}L_n(t)\right)$
805	6	.	to be replaced by	(note that $\operatorname{Re} L_n(t) = \log\vert\tau(t)\vert \to -\infty$ as $t \to a, b$).
806	-5	a 2π	to be replaced by	a continuous 2π
806	-4	$[2k\pi, 2(k+1)\pi]$	to be replaced by	$]2k\pi, 2(k+1)\pi[$
808	-8	connected.	to be replaced by	connected (may be empty).
808	-7	.	to be added	In particular, if K_1 and K_2 are polynomially convex and disjoint, then $K_1 \cup K_2$ is polynomially convex.
816	-6	$G_{1,\ldots,N}$	to be replaced by	$G_{\{1,\ldots,N\}}$
819	5	a contradiction to	to be replaced by	contradicts
820	-5	γ_j	to be replaced by	γ_j in Ω
821	-1	homologuous	to be replaced by	homologous
844	footnote 270		misplaced to page 846	
850	-7	$0 < \vert\eta\vert$	to be replaced by	$\inf_{\mathbb{D}} \vert f\vert < \eta$
851	-13	$N(f)$	to be replaced by	$Z(f)$
857	7	Remark	to be replaced by	Proposition
867	-3	*open*	to be replaced by	open
873	-2		delete the spurious ! symbol	
887	-14	.	to be replaced by	(note that x_0 is not an isolated point).
901	-15	[188]	to be replaced by	[p. 113, Chapter VI, Theorem 4.4] Whyburn
910	-16	endpoints	to be replaced by	endpoint
933	3	is path	to be replaced by	is a path
946	-7	$w_0 \in \Omega_1$	to be replaced by	$w_0 \in \partial\Omega_1$
955	-5	homeomorphims	to be replaced by	homeomorphisms
955	-1	.	to be replaced by	extending h.
956	2	$\mathbb{C}$ the target	to be replaced by	$\mathbb{C}$ is the target
956	17	.	add footnote	One may also directly consider the function $f - f(0)$.
965	5	$= = 0 = v(x_0)$	delete the second =	
967	4	Theorem	to be replaced by	Lemma

page	line			
967	9	$\prod_{k=1}^{n}$	to be replaced by	$\prod_{k=0}^{n}$
968	5	Theorem	to be replaced by	Lemma
981	-15	converge locally	to be replaced by	converge (with respect to the Euclidean norm on S_2) locally
981	-12	$\|\| \cdot \|\|$	to be replaced by	$\|\| \cdot \|\|_2$
981	-12	.	to be replaced by	(see Appendix 36).
982	9	$f(\iota_2(\iota_1(n)))$	to be replaced by	$(f_{\iota_2(\iota_1(n))})$
984	-3	no	to be replaced by	not a
991	$3, 4$		put into displaystyle	$\lambda(z) \leq \lambda_{\mathbb{D}}(z)$
991	-3	dz	to be replaced by	$\|dz\|$
992	9	map $f_r(z)$	to be replaced by	maps $f(z)$
1001	-9	Exemple	to be replaced by	Example
1001	-6	U	to be replaced by	$U \subseteq \mathbb{R}^n$
1003	-10	$= b$	to be replaced by	$= b + 1$
1005	10	iii)	to be replaced by	(3)
1016	-7	by	to be replaced by	by Lemma 2.1 (for $j = n + 2$) and
1016	-7	, there	to be replaced by	(Definition 17.4), there
1020	$-2, -5, -7$	$\mathbb{K}$	to be replaced three times by	$\mathbb{R}$
1028	3	Step, 1,	to be replaced by	Step 1,
1038	-2	the metric	to be replaced by	the standard metric
1040	19	Lemma	to be replaced by	Example
1042	-9			The proof environment was inadvertently shifted by Latex to the next page
1045	3			The proof symbol has been misplaced by Latex
1049	12	[102, 300]	to be replaced by	[102, p. 300]
1050	4	1	to be replaced by	$\frac{1}{2}$
1061	-5	$\bigcup_{k=1}^{n}$	to be replaced by	$\bigcup_{k=1}^{n-1}$
1079	5	(5)	to be replaced by	(1)

page	line			
1085	3	done.	to be replaced by	done, since each prime number p (equivalently positive irreducible number) is a prime element: in fact, suppose that p divides ab, say $pc = ab$, but not a. Then a and p have no proper common divisor, and so by the Euclidean algorithm, $1 = na + mp$ for some $n, m \in \mathbb{Z}$. Hence $b = n(ab) + (mb)p = n(cp) + (mb)p = (nc + mb)p$. Thus p divides b.
1085	9	irreducible factors	to be replaced by	prime elements
1085	14	Since	to be replaced by	Since by Observation 19.24
1086	$9 + 10$	between line 9 and 10	to be added	If R is an UFD, then (2) and (3) are equivalent.
1086	-13	(2) and (3) are still equivalent, but	to be deleted	
1088	11	$I_R(a, b)$	to be replaced by	$I := I_R(a, b)$
1095	$3, 4$	delete the sentence "Since...22.7)."		
1095	9	4.15	to be replaced by	4.14
1098	17	$\mathbf{u} \in C^\infty(U)$	to be replaced by	$\mathbf{u} \in C^\infty(U)^n$
1100	2	$\mathbf{u} \in C^\infty(U)$	to be replaced by	$\mathbf{u} \in C^\infty(U)^n$
1101	-15	Theorem 24.5	to be replaced by	Proposition 20.8
1106	12	Then	to be replaced by	If $\overline{A}$ is the uniform closure of A, then
1106	15	$\mathbf{1}$	to be replaced by	1
1107	-1	$\mathbf{1}$	to be replaced by	1
1108	12	$\mathbf{1}$	to be replaced twice by	1
1108	-6	Poposition	to be replaced by	Proposition
1113	-11, f.n. 379,380	strictly	to be replaced three times by	strongly
1125	8	$\mathbf{1}$	to be replaced twice by	1
1130	8	space X,	to be replaced twice by	space,
1131	11	$m(\mathbf{1})$	to be replaced by	$m(1)$
1133	-13	$m(\mathbf{1})$	to be replaced by	$m(1)$
1142	-4	$\mathbf{1}$	to be replaced twice by	1

page	line			
1148	$9, 10$	Corollary	to be replaced twice by	Theorem
1148	-1	Proposition	to be replaced by	Remark
1154	8	and that $a_n \neq 0$	to be deleted	
1182	8	no	to be replaced by	not a
1190	-2	dos	to be replaced by	does
1191	-10	$\left(\bigcap_{k=1}^{\infty} M^{\odot k}\right) =$	to be replaced by	$\left(\bigcap_{k=1}^{\infty} M^{\odot k}\right) \subseteq$
1196	-16	notationm	to be replaced by	notation
1196	-5	$\mathbf{c}(\mathbf{f} + \mathbf{a}g)$	to be replaced by	$\mathbf{c} \cdot (\mathbf{f} + \mathbf{a}g)$
1202	7	$\mathbf{r} \cdot \mathbf{x} = 1$	add footnote at 1	where 1 is the unit element in $\mathcal{R}$
1202	$7, 9, 10, 11$	$\mathbf{x}, x_j$	to be replaced by	$\tilde{\mathbf{x}}, \tilde{x}_j$
1209	5	$fu + t$	to be replaced by	$fu + t \cdot \mathbf{1}$
1213	4	$, \cdots ,$	to be replaced by	$, \dots ,$
1216	-10	reached	to be replaced by	achieved
1217	19	reducible	to be replaced by	*reducible*
1218	$1-6$	Proof of Corollary 23.43	to be replaced by	The case $n = 1$ has been done in Proposition 9.46. So it suffices to show that sufficient small perturbations of each $(a, \mathbf{b}) \in QI_{n+1}(A)$ belong again to $QI_{n+1}(A)$. This is obvious though in view of Proposition 23.24 and the fact that $U_n(A_u)$ is open by Proposition 7.322.
1221	9	bsr $I =$	to be replaced twice by	bsr $I \leq$
1221	-16	with 23.13	to be replaced by	with Definition 23.13
1236	8	poof	to be replaced by	proof
1250	$13/14$	to be added	at the end of the proof	As each z_k is a removable singularity of the k-th summand S_k, we deduce that the series converges locally uniformly in $\mathbb{C}$. Moreover, for each z_j, $f(z) = w_j \frac{W(z)}{(z-z_j)W'(z_j)} \left(\frac{z}{z_j}\right)^{n_j} + W(z)R_j(z)$, for some function R_j meromorphic in $\mathbb{C}$ with poles exactly at every $z_k, k \neq j$. Since $W(z_j) = 0$ and $\lim_{z \to z_j} \frac{W(z)}{(z-z_j)W'(z_j)} = 1$, we conclude that $f(z_j) = w_j$ for every $j \in \mathbb{N}$.
1251	9	Theorem	to be replaced by	Proposition
1253	6	Theorem	to be replaced by	Proposition

page	line			
1254	-15	Theorem	to be replaced by	Proposition
1258	15	Proposition	to be replaced by	Theorem
1261	-13	$\mathbb{C}[x_1, \ldots, x_n]$	to be replaced by	$\mathbb{C}[z_1, \ldots, z_n]$
1268	2	$\dim_c X$	to be replaced twice by	$\dim X$
1271	16	τ_b	to be replaced by	τ_d
1271	-10	$= t_n < s_n$	to be replaced by	$\leq t_n < s_n$
1275	-15	g	to be replaced by	$\check{g}$
1281	$14, 16$	$\mathbb{K}$	to be replaced twice by	$\mathbb{C}$
1285	-8	Lemma	to be replaced by	Corollary
1289	6	identify	to be replaced by	identify the n-tuple
1292	-4	$\mathbb{K}^n$	to be replaced by	$\mathbb{K}$
1293	5	$j \geq n+1$	to be replaced by	$j \geq n+2$
1299	9	sse	to be replaced by	see
1304	9	e_1	to be replaced by	$\mathbf{e}_1$
1307	$-2, -1$		delete the last sentence	~~thus....disconnected~~
1313	-9	$L^1[0,1]$	to be replaced by	$L^1([0,1])$
1320	-13	Theorem	to be replaced by	Lemma
1327	-4	, proof	to be replaced by	proof
1331	12	$\frac{c_3(\xi)}{(z-\xi)^3} + \frac{c_4(\xi)}{(z-\xi)^4} + \cdots$	to be replaced by	$\left\lvert \frac{c_3(\xi)}{(z-\xi)^3} + \frac{c_4(\xi)}{(z-\xi)^4} + \cdots \right\rvert .$
1345	-1	14.11	to be replaced by	Theorem 14.11
1356	8	26.19	to be enlarged by	applied to the set $\{z \in \mathbb{C} : \lvert z+1 \rvert \leq 1\} \cup \{z \in \mathbb{C} : \lvert z-2 \rvert \leq 2\}$
1362	-15	7.129	to be replaced by	Example 7.129
1363	-14	z	to be replaced three times by	Z
1373	12	Lemma 26.55	to be replaced by	Proposition 26.55
1373	13	$f/(\prod_{j=1}^p$	to be replaced by	$f/\prod_{j=1}^p$
1373	-12		to be added	Moreover, $\partial x_j \in C^1(K^\circ)$.
1385	15	Theorem 17.17	to be replaced by	Proposition 17.17

page	line			
1389	7	Theorem	to be replaced by	Proposition
1395	1	Rubel de Boer	to be replaced by	Friedland-den Boer-Rubel
1405	17	due to compactness	to be replaced by	by definition of $\mathscr{H}(K)$
1407	-1	$e^{(2\pi i \arg a)(k/m)}$	to be replaced by	$e^{i(\arg a+2k\pi)/m}$
1423	-1	$\lvert c\rvert + \lvert d\rvert \neq 0$	to be replaced by	$ad - bc \neq 0$
1424	-6	If we denote by ψ the	to be replaced by	Let ψ be the
1424	-5	, then	to be replaced by	, and which is given by
1443	-6	$x \mapsto$	to be replaced by	for $0 \leq a < 1$, $x \mapsto$
1444	11	, we	to be replaced by	whenever $0 \leq a < 1$, we
1453	9	Proposition	to be replaced by	Lemma
1463	-19	$\alpha \notin E \cap \mathbb{T}$	to be replaced by	$\alpha \in \mathbf{D} \setminus E$
1480	-9	e^b	to be replaced by	e^a
1486	-7	Remark	to be replaced by	Proposition
1507	14	defined	to be replaced by	given
1519	6	Since	to be replaced by	Since by Lemma 6.50
1536	8	Proposition	to be replaced by	Lemma
1536	-6	Proposition	to be replaced by	Theorem
1540	-8	3	to be replaced by	$3n$
1540	-1	Λ	to be replaced by	$\frac{\Lambda}{2}$
1540	-1	$C_0(\delta, n)$	to be replaced by	$\frac{1}{2}\, C_0(\delta, n)$
1546	-1	Hence	to be replaced by	Hence, by Schark's Theorem 27.15,
1553	-5	h	to be replaced twice by	f
1573	4	be	to be replaced by	be a
1580	-10	$\lvert\widehat{b_j}\rvert$	to be replaced by	$\lvert\, \widehat{b}\, \rvert$
1581	-8	Theorem	to be replaced by	Proposition

page	line			
1583	-8	But	to be replaced by	Note that
1583	-7	So	to be replaced by	Thus $\|\|g\|\|_\infty$ cannot be 1, since the only point $w \in \mathbb{T}$ where $\|1+\overline{\alpha}w\|/2 = 1$ equals $w = \alpha \neq 1$, but $p \neq \alpha$. Hence $\|\|g\|\|_\infty < 1$. But
1583	-6	$= \left\|\frac{1+\overline{\alpha}p(x_0)}{2}\right\| = \left\|\frac{1+\overline{\alpha}}{2}\right\| < 1$	to be deleted	
1609	-4	$1 - r^2 - 2r\cos s$	to be replaced by	$1 + r^2 - 2r\cos s$
1631	-7	$h \in A$	to be replaced by	$\mathbf{h} \in A^n$
1661	-5	A_2,	to be replaced by	$A_2 = C(\mathbf{D}, \mathbb{C})$,
1662	-3	semigroup	to be replaced by	sub-semigroup
1665	-9	$g(0) \neq 0$; hence,	to be deleted	
1665	-8	.	to be replaced by	, because $U_n(A)$ is an open set (Proposition 7.322).
1671	-14	basis	to be replaced by	disk
1682	-5	*Hadamard multiplication*	to be replaced by	Hadamard multiplication
1694	-9	$C(X, A^n)$	to be replaced by	$C(X, A)^n$
1700	5	is totally	to be replaced by	is compact and totally
1705	12	1	to be replaced by	**1**
1706	8	. Denote	to be preceded by	If $\sigma_A(a_n) = \{0\}$ then, for all $\varepsilon > 0$, $a_n - \varepsilon \cdot \mathbf{1}_A \in A^{-1}$, and so $(a_1, \ldots, a_n - \varepsilon \cdot \mathbf{1}_A) \in U_n(A)$ is an invertible approximant of **a**. If $\sigma_A(a_n) \neq \{0\}$, then we proceed as follows. Denote
1706	9	, and	to be replaced by	, $\lambda \neq 0$, and
1707	-12	Theorem	to be replaced by	Corollary
1711	-16	by the proof of Lemma	to be replaced by	by Lemma
1729	-13	$U(B)$	to be replaced by	$U_1(B)$
1734	10	Theorem 7.17	to be replaced by	Lemma 7.120
1741	$-17, -15$	Theorem	to be replaced twice by	Lemma
1754	14	1	to be replaced by	**1**
1756	$-12, -2$	Proposition	to be replaced twice by	Corollary
1790	7	Example 34.43	to be replaced by	Proposition 34.43
1832	9	square	to be replaced by	rectangle

page	line			
1843	-10	from	to be replaced by	using
1848	-12	.	to be added by	Since $(F,G) = (f_2,g_2)\begin{pmatrix} a & g_1 \\ b & -f_1 \end{pmatrix}$ for an invertible matrix whose determinant is -1, we deduce from the invertibility of the pair (f_2,g_2) that $(F,G) \in U_2(A(\mathbb{D})_{\mathrm{sym}})$ (Observation 7.321).
1864	-1	Theorem	to be replaced by	Proposition
1868	13	but not	to be replaced by	or
1868	14	bsr $C(K,\mathbb{C})_{\mathrm{sym}} = 1$ (Corollary 34.20). Hence, by	to be replaced by	by Corollary 34.54
1868	-1	hat	to be replaced by	that
1871	3	$g_j :=$	to be deleted	
1878	11	$\mathcal{T}_A$	to be replaced by	$\mathcal{T}_{AP}$
1879	8	Let θ be	to be replaced by	Let $\theta > 0$ be
1879	20	$k_j \in \mathbb{N}$	to be replaced by	$k_j \in \mathbb{Z}$
1893	4/1	by Lemma 35.23	to be moved to line 1	then,....
1893	7	.	to be replaced by	. Recall that $x_1 = u_1$.
1896	6	minumum	to be replaced by	minimum
1896	6	.	to be replaced by	. Due to the boundedness of f, the limit ℓ is finite.
1897	-10	the	to be replaced by	an
1897	-9	.	to be replaced by	, where $\inf_{x\in\mathbb{R}} \lvert f(x+t_n) - f(x)\rvert \le \varepsilon$.
1898	2	I_n.	to be replaced by	t_n.
1900	3	.	to be replaced by	In fact, let $E_n = \{\lambda \in \mathbb{R} : \widehat{q}_n(\lambda) \neq 0\}$. Then $E := \bigcup_{n=1}^{\infty} E_n$ is countable and $\widehat{f}(\lambda) = 0$ outside E.
1914	13	Theorem	to be replaced by	Proposition
1914	-7	yields	to be replaced by	yields item (1) of
1914	-6	:	to be replaced by	, whereas (2) is an immediate consequence of (1).
1915	$1-3$		replace the sentence "By..." with	For the rest of this chapter we identify the subset $\tau(\mathbb{R})$ of $M(\mathrm{AP})$ with $\mathbb{R}$.
1919	7–11	We claim...on E^+.	to be replaced by	As $\mathbb{R}^+ \subseteq E^+$, we obviously have that $f \equiv 0$ on $\mathbb{R}^+$.
1919	$-12/-11$			Interchange x_n with y_n and vice-versa
1925	2/3	$\lim\limits_{T\to\infty}$	to be replaced twice by	$\limsup\limits_{T\to\infty}$

page	line			
1929		Lemma	to be replaced by	Corollary
1932	-12	positive	to be replaced by	non-negative
1939	-3	35.72(2)	to be replaced by	35.72(1)
1942	-10	Theorem 25.6 (6)	to be replaced by	Corollary 25.6 (7)
1947	-4	1.99	to be preceded by	Example 1.99
1950	12	Theorem	to be replaced by	Observation
1966	16	matrix	to be replaced by	matrix (with determinant 1)
1968	-15	q_j, q_j	to be replaced by	p_j, q_j
1973	17	ϕ	to be replaced by	Φ
2006	-6	uniquness	to be replaced by	uniqueness
2007	3	*Weierstrass factors*	to be replaced by	Weierstrass factors
2007	11	$\log(1-z) = \sum_{n=1}^{\infty}$	to be replaced by	$\log(1-z) = -\sum_{n=1}^{\infty}$
2007	4, 13	root	to be replaced twice by	zero
2018	1	$\mathbb{C} \setminus \mathbb{K}$	to be replaced by	$\mathbb{C} \setminus K$
2018	15	.	to be replaced by	and $\overline{U}$ is homeomorphic to $\mathbf{B}_n = \overline{B}_n$.
2019	7	Corollary	to be replaced by	Lemma
2078	12	$\lambda(u+v)$	to be replaced by	$\lambda \cdot (u+v)$
2031	$-3/-2$	and...(1,0).	to be replaced by	Since $f_y(1,0) = 0$, we cannot use the implicit function theorem. But by substituting $u = y^2$, we obtain $F(x,u) = (x-1)^2 + u - \kappa^2(1-\sqrt{x^2+u})^2 = 0$, and $F_u(1,0) = 1 \neq 0$. So there is $u \in C^1(U)$ for a neighborhood U of 1 with $F(x,u(x)) = 0$. Now $u(x) \geq 0$ for $x \in U, x \leq 1$ (note that $u(1) = u'(1) = 0$ and $u''(1) = 2(\kappa^2-1) > 0$). Hence $y(x) := \sqrt{u(x)}$ is a solution.
2032	2	$y'(t)$	to be replaced by	$y'(x(t))$
2032	-10	$y'(0)$	to be replaced by	$y'(1)$
2055	-7	$\|(f \circ \gamma)'(t)$	to be replaced by	$\|(f \circ \gamma)'(t)\|$
2062	-9	$\max_{\xi\in\gamma}\{\|u(\xi) - u(\xi_0)\|$	to be replaced by	$\max_{\xi\in\gamma} \|u(\xi) - u(\xi_0)\|$

page	line			
2071	-13	in	to be replaced by	of
2071	-5	the fact	to be replaced by	Observation
2122	-1	$\frac{1}{2\pi i}$	to be replaced by	$\frac{m!}{2\pi i}$
2123	10	homologuous	to be replaced by	homologous
2126	cell -7	$A(S,K)\ S \subseteq K \subseteq \mathbb{C}$	to be replaced by	$A(S,K),\ S \subseteq K \subseteq \mathbb{C}$
2127	cell -1	to dis.	to be replaced by	to. dis.
2128	cell 3	$\mathbb{Z}/m\mathbb{Z} = 1$	to be replaced by	$\mathbb{Z}/m\mathbb{Z}$
2128	cell 12	comp..,	to be replaced by	comp.,
2129	-2	Let...exist	to be replaced by	For which free ultrafilters $\mathcal{F}$ on $\mathbb{N} = \{0, 1, 2, \dots\}$ does there exist
2129	-1	$f \in \mathcal{F}$	to be replaced by	$F \in \mathcal{F}$
2129			to be added after last line	Concerning this question, we were informed by Gerd Herzog that some, but not all ultrafilters on $\mathbb{N}$ have this property This is on page 76 and 343-344 in [**?**].
2131	1/2	Corollary 4.60	to be replaced by	Theorem 4.60
2134	-5	in 27.192	to be replaced by	in Proposition 27.192
2137	-8	Propositions 1.171	to be replaced by	Proposition 1.171
2146	-17	topolgy	to be replaced by	topology
2173	8	$L^1[01,]$	to be replaced by	$L^1[0,1]$

Additional material

to be added on page 12

Proposition 0.1. *Let X be a topological space and suppose that $A \subseteq X$ is closed. Then*

(1) $\left(\overline{A^\circ}\right)^\circ = A^\circ$.

(2) $\partial A^\circ = \partial \overline{A^\circ}$.

Although (1) and (2) are equivalent for open sets G, neither of them may hold in that case.

Proof. (1) Just use that

$$A^\circ \subseteq A \Longrightarrow \overline{A^\circ} \subseteq \overline{A} = A \Longrightarrow \left(\overline{A^\circ}\right)^\circ \subseteq A^\circ \subseteq \left(\overline{A^\circ}\right)^\circ.$$

(2) By (1),

$$\partial \overline{A^\circ} = \overline{A^\circ} \setminus \left(\overline{A^\circ}\right)^\circ = \overline{A^\circ} \setminus A^\circ = \partial A^\circ.$$

(2) is equivalent to (1) by the preceding Proposition **??**. If $G = \mathbb{C} \setminus \{0\}$, then $\partial G = \{0\}$, but $\partial \overline{G} = \emptyset$. □

T be adde on page 73.

In view of Lemma **??** one may ask whether under the assumptions of Theorem **??** one actually has $\emptyset \neq \partial C \subseteq \partial A$?

Proposition 0.2. *Let X be a compact connected Hausdorff space, A a proper closed nonvoid subset and C a connected component of A. Then the following statements are true:*

(1) *There exists such a triple (X, A, C) such that one does not have $\partial C \subseteq \partial A$.*
(2) *If, additionnally, X admits a topological basis of connected sets, then we have*
$$\emptyset \neq \partial C \subseteq \partial A.$$

Proof. (1) See figure 1, where A is the union of the red rectangles and the dotted red line C, and where X is the union of A with the black line L. Here $\partial A = \partial_{\mathbb{C}} A \cap L$ and $\partial C = C$. Note that $C \cap \partial A = \{0\}$. □

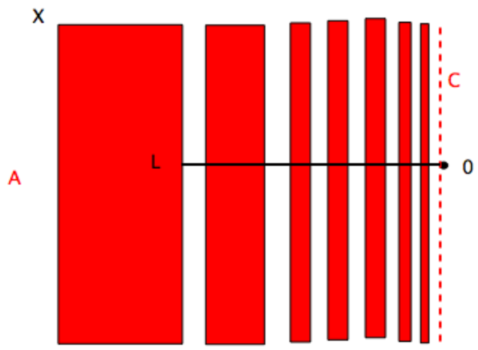

FIGURE 1. bumping theorem

(2) This is similar to the proof of Lemma **??**. By Proposition **??**, C is closed. Moreover, $\partial C \neq \emptyset$, since in connected spaces $\partial M = \overline{M} \setminus M^\circ \neq \emptyset$ for any proper non-void set M. Let $x_0 \in \partial C$ and let W be any connected open set containing x_0 . Then $W \cap A^c \neq \emptyset$, since otherwise $W \subseteq A$ and so $W \cup C$ would be a connected set strictly bigger than C (as W intersects C^c) and contained in A, a contradiction to the maximality of C. Since $x_0 \in \partial C \subseteq C \subseteq A$, we conclude that $x_0 \in \partial A$.

Example 1.176. *Each convex open set in $\mathbb{C}$ is a simply connected domain.*

Proof. Let $G \neq \mathbb{C}$ be open and convex. Obviously G is (path)-connected. Suppose that $\mathbb{C} \setminus G$ admits a bounded component K. Let $a \in K$. Using if necessary a translation, we may assume that $a \in \mathbb{R}$. As K is bounded, there is a point $b \in K$ with minimal real part (and $b \leq a$). Then $]-\infty, b[$ cannot be entirely contained in $\mathbb{C} \setminus G$, since otherwise $]-\infty, b[\, \cup K = \,]-\infty, b] \cup K$ would be a connected subset of $\mathbb{C} \setminus G$, destroying the maximality of K. Hence, there is a point $x_0 \in G \cap \mathbb{R}$ at the left of $b \leq a$. Similarly, there is a point $x_1 \in G \cap \mathbb{R}$ at the right of a. Thus G cannot be convex, as $[x_0, x_1]$ is not contained in G. We conclude that G is a simply connected domain. □

on page 77, proof of Example 1.176 to be replaced by:

Let $G \neq \mathbb{C}$ be open and convex. Obviously G is (path)-connected. Suppose that $\mathbb{C} \setminus G$ admits a bounded component K. Since K is compact, there is a point $b \in K$ with minimal real part. Using, if necessary a translation, we may assume that $b \in \mathbb{R}$. Then $]-\infty, b[$ cannot be entirely contained in $\mathbb{C} \setminus G$, since otherwise $]-\infty, b[\, \cup K = \,]-\infty, b] \cup K$ would be a connected subset of $\mathbb{C} \setminus G$, destroying the maximality of K. Hence there is a point $x_0 \in G \cap \mathbb{R}$ at the left of b. Similarly, there is a point $x_1 \in G \cap \mathbb{R}$ at the right to b. Thus G cannot be convex as $[x_0, x_1]$ is not contained in G. We conclude that G is a simply connected domain.

on page 268 add the following section:

The Bergman representation

One easily deduces from Cauchy's integral formula in Theorem?? 4.15 that for $f \in A(\mathbf{D})$

$$f(z) = \frac{1}{2\pi i} \int_{|\xi|=1} \frac{f(\xi)}{\xi - z}\, d\xi.$$

In this section we present its counterpart for planar integrals.

Lemma 0.3. *Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ have absolutely convergent power series; that is $\sum_{n=0}^{\infty} |a_n| < \infty$. Then*

$$f(z) = \frac{1}{\pi} \iint_{\mathbb{D}} \frac{f(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta). \tag{0.1}$$

and

$$\overline{f(0)} = \frac{1}{\pi} \iint_{\mathbb{D}} \frac{\overline{f(\zeta)}}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta). \tag{0.2}$$

Proof. Using that

$$\frac{1}{(1-\overline{\zeta}z)^2} = \sum_{n=0}^{\infty} (n+1)\overline{\zeta}^n z^n,$$

we obtain (due to the absolute convergence)

$$\frac{f(\zeta)}{(1-\overline{\zeta}z)^2} = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_n \zeta^n (k+1)\overline{\zeta}^k z^k.$$

Since this double series converges for each fixed $z \in \mathbb{D}$ (absolutely) and uniformly in ζ, we have $\iint \sum\sum = \sum\sum\iint$. Now

$$\iint_{\mathbb{D}} \zeta^n\overline{\zeta}^k d\sigma_2(\zeta) = \int_{r=0}^{1}\int_{\theta=0}^{2\pi} r^{n+k} e^{i(n-k)\theta} r\,d\theta\,dr = \begin{cases} \displaystyle\int_0^1 r^{2n} 2\pi r\,dr = \frac{\pi}{n+1} & \text{if } k = n \\ 0 & \text{if } k \neq n. \end{cases}$$

Hence, as f is integrable on $\mathbb{D}$, we may apply Fubini's theorem (Appendix **??** (4)) on iterated integrals, to conclude that

$$\frac{1}{\pi}\iint_{\mathbb{D}} \frac{f(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta) = \sum_{n=0}^{\infty} a_n z^n = f(z),$$

hence (1) holds. To prove (2), it suffices to use that

$$\iint_{\mathbb{D}} \overline{\zeta}^n\overline{\zeta}^k d\sigma_2(\zeta) = \int_{r=0}^{1}\int_{\theta=0}^{2\pi} r^{n+k} e^{-i(n+k)\theta} r\,d\theta\,dr = \begin{cases} \displaystyle\int_0^1 2\pi r\,dr = \pi & \text{if } k = n = 0 \\ 0 & \text{if } k + n > 0. \end{cases}$$

□

As a corollary, we obtain the following nice formula.

Corollary 0.4. *Let $a \in \mathbb{D}$. Then*

$$\frac{1}{\pi}\iint_{\mathbb{D}} \frac{(1-|a|^2)^2}{|1-\overline{a}z|^4}\, d\sigma_2(z) = 1.$$

Proof. Just apply Lemma 0.3 to

$$f(z) = (1-\overline{a}z)^{-2} = \sum_{n=0}^{\infty}(n+1)\overline{a}^n\, z^n$$

which has absolutely convergent power series. □

As we are going to show, the formula in Lemma 0.3 is valid for all holomorphic functions f for which $\sum_{n=0}^{\infty}\frac{|a_n|^2}{n+1} < \infty$ (or equivalently for $\iint_{\mathbb{D}} |f|^2\, d\sigma_2 < \infty$), or more generally for those f for which $\iint_{\mathbb{D}} |f|\, d\sigma_2 < \infty$. These are the so-called Bergman functions.

Theorem 0.5 (Bergman representation)**.** *Let $f \in H(\mathbb{D})$ satisfy $||f||_1 := \iint_{\mathbb{D}} |f|\, d\sigma_2 < \infty$. Then*

$$f(z) = \frac{1}{\pi}\iint_{\mathbb{D}} \frac{f(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta). \tag{0.3}$$

and

$$\overline{f(0)} = \frac{1}{\pi}\iint_{\mathbb{D}} \frac{\overline{f(\zeta)}}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta). \tag{0.4}$$

Proof. Let f_r be defined by $f_r(z) = f(rz)$, $z \in \mathbb{D}$. Then f_r has absolute convergent power series and so, by Lemma 0.3,

$$f_r(z) = \frac{1}{\pi}\iint_{\mathbb{D}} \frac{f_r(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta).$$

Now $||f_r - f||_1 \to 0$ (see below). So, as the numerator is bounded away from zero by $(1-|z|)^2$ for fixed z,

$$\iint_{\mathbb{D}} \frac{f_r(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta) \to \iint_{\mathbb{D}} \frac{f(\zeta)}{(1-\overline{\zeta}z)^2}\, d\sigma_2(\zeta).$$

Identity (0.3) now follows. This norm approximation is straightforward though [1]. In fact,

$$\begin{aligned}
||f_r - f||_1 &= \iint_{|z|\le\eta} |f_r(z)-f(z)| d\sigma_2(z) + \iint_{\eta\le|z|<1} |f_r(z)-f(z)| d\sigma_2(z)\\
&\le \iint_{|z|\le\eta} |f_r(z)-f(z)| d\sigma_2(z) + \iint_{\eta<|z|\le1} (|f_r(z)|+|f(z)|) d\sigma_2(z)\\
&=: I_1(\eta) + I_2(\eta)
\end{aligned}$$

Since $||f||_1 < \infty$, and $\lim_{\eta\to1}\iint_{|z|\le\eta}|f| d\sigma_2 = ||f||_1$, we conclude that the integral $\int_{\eta_1\le|z|<1}|f| d\sigma_2$ is less that $\varepsilon/4$ whenever η_1 is close to 1. Now let r_0 and η_0 be so close to 1 that $r_0\eta_0 \ge \eta_1$. Then for all $r \in [r_0, 1[$, $I_2(\eta_0) \le \varepsilon/2$. For this η_0 the first integral $I_1(\eta_0)$ is less than $\varepsilon/2$ whenever $r \ge r_1 \ge r_0$ is close to 1 (due to uniform convergence of the integrand on $|z| \le \eta_0$).

---

[1] We follow here [?].

□

An entirely different proof, based on Hilbert space methods, is for instance in [**?**] or [**?**].

On page 365, add the following, as Lemma 6.50:

Lemma 0.6. *For $a, b \in \mathbb{D}$, let $L_a(z) := \dfrac{a-z}{1-\overline{a}z}$ and similarily for L_b. Then*

$$|L_a(z) - L_b(z)| \leq \frac{4\,|a-b|}{(1-|a|)(1-|b|)}.$$

Proof. Just estimate:

$$\begin{aligned}
|L_a(z) - L_b(z)| &= \left|\frac{a-z}{1-\overline{a}z} - \frac{b-z}{1-\overline{b}z}\right| \leq \frac{|(a-z)(1-\overline{b}z) - (b-z)(1-\overline{a}z)|}{(1-|a|)(1-|b|)} \\
&= \frac{|(a-b) + z^2(\overline{b}-\overline{a}) + z(b\overline{a} - \overline{b}a)|}{(1-|a|)(1-|b|)} \\
&\leq \frac{2|a-b| + |b\overline{a} - \overline{b}a|}{(1-|a|)(1-|b|)} \\
&= \frac{2|a-b| + |b(\overline{a}-\overline{b}) - (a-b)\overline{b}|}{(1-|a|)(1-|b|)} \\
&\leq \frac{4\,|a-b|}{(1-|a|)(1-|b|)}
\end{aligned}$$

□

The proof is exactly the same as that for the complex-valued functions (see Section 4.2). For (3), for instance, we refer to Appendix 104 and to Appendix 100 for (4).

On page 435, the three lines above are to be replaced by:

Proof. The proof for (1) is exactly the same as that for the complex-valued functions (see Theorem **??**). For (3), we refer to Appendix **??** and to Appendix **??** for (4). For (2), it suffices (by definition) to prove that the series $\sum_{n=0}^{\infty} a_n z^n$ converges uniformly and absolutely (that is $\sum_{n=0}^{\infty} ||a_n||\,|z|^n < \infty$) on each closed disk $\overline{D}(0,\rho')$ with $0 < \rho' < \rho$ and diverges for every $z \in \mathbb{C}$ with $|z| > \rho$ in the sense that $(a_n z^n)$ is not a zero-sequence in A. The latter case is obvious, since $1/|z| < 1/\rho = \limsup_n \sqrt[n]{||a_n||}$ implies for every $N \in \mathbb{N}$ the existence of $n \geq N$ such that $\frac{1}{|z|} < \sqrt[n]{||a_n||}$. Hence $1 < ||a_n z^n||$. In the first case, let ρ^* satisfy $\rho' < \rho^* < \rho$, and choose $N \in \mathbb{N}$ so that

$$\sup\left\{ \sqrt[n]{||a_n||} : n \geq N \right\} < \frac{1}{\rho^*}.$$

Hence, for every $z \in D(0,\rho')$ and $n \geq N$,

$$||a_n z^n|| \leq ||a_n||(\rho')^n = ||a_n||(\rho^*)^n \left(\frac{\rho'}{\rho^*}\right)^n \leq \left(\frac{\rho'}{\rho^*}\right)^n =: \delta^n.$$

This implies the absolute and uniform convergence of the A-valued series on $D(0,\rho')$ since $0 < \delta < 1$ and

$$\left\| \sum_{n=L}^{M} a_n z^n \right\| \leq \sum_{n=L}^{M} ||a_n z^n|| \leq \sum_{n=L}^{M} \delta^n.$$

□

on page 785, add:

Remark 0.7. (1) Whereas all three properties in Theorem **??** 10.11 hold for $\mathbb{R}^n$ endowed with the Euclidean norm, (due to Brouwer's fixed point theorem, Theorem **??**), none of them holds in infinite dimensional normed spaces (see [**?**]). A simple example [2] of a fixed point-free map on the ball can be given in

$$\ell^1(\mathbb{N}) = \{x = (x_0, x_1, \dots) \in \mathbb{R}^{\mathbb{N}} : ||x|| = \sum_{j=0}^{\infty} |x_j| < \infty\}.$$

Let $f((x_n)_{n\in\mathbb{N}}) := (1 - ||x||, x_0, x_1, \cdots)$. Then

$$||f(x) - f(y)|| = \big|||y|| - ||x||\big| + ||x - y|| \leq 2||x - y||.$$

Hence f is continuous, and for $||x|| \leq 1$,

$$||f(x)|| = 1 - ||x|| + ||x|| = 1.$$

So f has no fixed point in the open unit ball, and neither on the unit sphere S because $f(x) = x$ implies that $0 = 1 - ||x|| = x_0$ and $x_j = x_{j+1}$ for all $j \in \mathbb{N}$. Hence all the coordinates coincide with 0, but $\mathbf{0} \notin S$.
(2) On the other hand, all properties in Theorem **??** hold for arbitrary real normed spaces E of finite dimension n. To see this, first note that $(E, ||\cdot||)$ is topologically isomorphic via a linear map ϕ to $(\mathbb{R}^n, ||\cdot||_2)$, (Corollary 7.25 **??**). Let B be the closed unit ball in E. Then $U := \phi(B^\circ) = \phi(B)^\circ$ obviously is a bounded open convex set in $\mathbb{R}^n$. Note that $\phi(B) = \overline{U}$. Hence, by Appendix 19 **??**, $\phi(B)$ is homeomorphic via a map ϕ_n to the closed Euclidean ball $\mathbf{B}_n$. Thus, B is homeomorphic to $\mathbf{B}_n$. Since the fixed-point property is invariant under homeomorphisms, we conclude from Theorem **??** that every continuous self-map of B has a fixed point.

Another way to see this, is to use Proposition **??** 16.2, telling us that $\phi(B)$ is a retract, and to conclude from Theorem **??** 16.17, that $\phi(B)$ has the fixed-point property.

[2] This has been communicated to us by Gerd Herzog

on page 1287 add the following Example:

Example 0.8. *The function* $f : \frac{1}{2}\mathbb{T} \cup \mathbb{T} \to \mathbb{C}$ *given by* $f(z) = z$ *if* $|z| = 1/2$ *and* $f(z) = 1$ *if* $|z| = 1$ *does not have a zero-free continuous extension to the associated annulus* $\{1/2 \le |z| \le 1\}$.

Proof. We first write f in the form given by Eilenberg's Theorem [**?**, Theorem 12.20]. Let $a_1 = 0$ and $a_2 = 2/3$. Then a_1 and a_2 belong to two different holes of $K := \frac{1}{2}\mathbb{T} \cup \mathbb{T}$. Since a_2 belongs to the unbounded component of $|z| = 1/2$, by [**?**, Proposition 12.19], $z - a_2 = e^h$ for some $h \in C(\frac{1}{2}\mathbb{T})$. Since a_1 and a_2 belong to the same component of $\mathbb{C} \setminus \mathbb{T}$, we have that $z/(z - a_2) = e^k$ for some $k \in C(\mathbb{T})$ ([**?**, Lemma 12.14]). Thus, by putting $g = -k$ on $\mathbb{T}$ and $g = h$ on $\frac{1}{2}\mathbb{T}$, f writes as

$$f(z) = \frac{z}{z - 2/3} e^{g(z)}.$$

By [**?**, Definition 25.33], $1/2 < |z| < 1$ is an essential hole for f. Hence, by [**?**, Proposition 25.34], f does not admit a continuous zero-free extension to the annulus. □

A different proof (solely based on Corollary **??** to Brouwer's theorem) is given in [**?**].

p. 1553, lines 3-12 to be replaced by:

function $q(z) := \arg z = t$, where $z = e^{it}$, $-\pi < t \leq \pi$, does not belong to $H^\infty(\mathbb{T}) + \mathbb{C}(\mathbb{T})$. Note that q is continuous excepted at the point $z = -1$, where it has a jump. Suppose to the contrary that $q = u + v$ for $u \in H^\infty(\mathbb{T})$ and $v \in C(\mathbb{T})$. Then

$$z = e^{iq} = e^{iu}\, e^{-iv} := e^h e^k,$$

where $h \in H^\infty(\mathbb{T})$ and $k \in C(\mathbb{T})$. Hence $ze^{-h} = e^k$. Now the identity

$$P[e^k] = e^{P[k]}$$

is a consequence of the uniqueness of the Dirichlet problem, because both functions are harmonic in $\mathbb{D}$, continuous in $\overline{\mathbb{D}}$, and have the same boundary values e^k. Since the Poisson operator is multiplicative on $H^\infty(\mathbb{T})$ (Corollary **??**),

$$e^{P[k]} = P[e^k] = P[ze^{-h}] = zP[e^{-h}] = ze^{-P[h]} \in H^\infty(\mathbb{D})$$

(Proposition **??**), we see that $e^{P[k]}$ is holomorphic, too. Hence $P[k]$ is holomorphic (Proposition **??**). Consequently z would equal an exponential in $\mathbb{D}$; a contradiction.

On page 1644 one adds the following:

As a consequence we obtain the following two results.

Corollary 0.9. *Let $A \subseteq C(X, \mathbb{C})$ be a Banach function algebra over $\mathbb{C}$ on a compact Hausdorff space X. If every closed subset of X is a peak set (respectively weak peak set) for A, then $A = C(X, \mathbb{C})$.*

Proof. It obviously suffices to show this for weak-peak sets (since each peak set is a special weak peak set). In the Bade-Curtis theorem **??** we take $C = 1$ and $\alpha = 1/4$ and $||f|| := ||f||_\infty$. Let E, F be closed subsets of X with $E \cap F = \emptyset$. Since F is a weak peak set, there is $p \in A$ with $||p||_\infty = 1$, $p = 1$ on F and $\sup |p|_E < 1$ (Lemma **??**). By taking a sufficient high power $f := p^s$ of p, we conclude that $|f| < \alpha$ on E and $|f - 1| = 0 < \alpha$ on F. Thus, by Theorem **??**, $A = C(X, \mathbb{C})$. □

Theorem 0.10 (Izzo)**.** *Let $A \subseteq C(X, \mathbb{C})$ be a uniform algebra on a compact Hausdorff space X. Let μ be a positive Borel measure on X. Suppose that every closed subset E of μ-measure zero is a peak (respectively weak peak) set for A. Then E is an interpolation set.*

Proof. Consider the restriction algebra $A|_E$. By Corollary **??** (1), $A|_E$ is uniformly closed in $C(E, \mathbb{C})$, and so it is a uniform algebra, too. Now if $F \subseteq E$ is a closed subset of E, then $\mu(E) = 0$ implies $\mu(F) = 0$. By hypothesis, F is a peak (respectively weak peak) set for A. This obviously implies that F is a peak (respectively weak peak) set for $A|_E$. By Corollary **??**, $A|_E = C(E, \mathbb{C})$. In other words, E is an interpolation set. □

4) As $h^{-1}(y) = y/(1 + f(y))$, we deduce from the continuity of f (Lemma 7.14 (5)) that h and h^{-1} are continuous. □

On page 2019, one adds the following additional item:

5) We claim that the map $F : \overline{U} \to \mathbf{B}_n$ given by

$$F(x) = \frac{x}{1 - f(x) + ||x||}$$

is a homeomorphism with inverse

$$F^{-1}(y) = \frac{y}{1 - ||y||_2 + f(y)}.$$

In fact, by Lemma **??**, $0 \leq f(x) \leq 1$ and $f(0) = 0$. So F is well-defined and continuous as U is convex. Given $y \in \mathbf{B}_n$, that is $||y||_2 \leq 1$, we immediately deduce from Lemma **??** (2), (7) that with

$$x := \frac{y}{1 - ||y||_2 + f(y)},$$

$0 \leq f(x) \leq 1$, hence $x \in \overline{U}$, and $F(x) = y$.

on page 2037 add these few lines:

We also deduce that for every $\kappa > 0$, $D^*_\kappa \cap \{z \in \mathbb{C} : \operatorname{Re} z \geq 0\}$ is convex. In particular, we obtain the following analogon to Appendix **??**:

Appendix 0.11. *Let $z_j = r_j e^{it_j}$ be points in $\mathbb{C}$ with $0 < r_j < 1$ and $|t_j| < \pi/2$. Consider the triangle $\Delta = \langle z_1, z_2, 1 \rangle$; that is the closed convex hull of the three points $z_1, z_2, 1$. Then, for every $z = re^{it} \in \Delta$ with $|t| < \pi/2$,*

$$\frac{|t|}{1-r} \leq \max\left\{\frac{|t_1|}{1-r_1}, \frac{|t_2|}{1-r_2}\right\}.$$

on page 2046, add this new Appendix:

Appendix 0.12. *Given* $0 < r < 1$*, there exists a covering of* $[0,1[$ *with pseudohyperbolic disks* $D(x_n, r)$ *such that* $x_n < x_{n+1}$ *and* $r < \rho(x_n, x_{n+1})$ *(so that* x_{n+1} *does not belong to the closure of* $D_\rho(x_n, r)$ *but* $D_\rho(x_n, r) \cap D_\rho(x_{n+1}, r) \neq \emptyset$*).*

Proof. Let $x_0 := 0$. Then $x_n \to 1$ has to be chosen so that

$$P := \frac{x_n + r}{1 + x_n r} \in D_\rho(x_{n+1}, r), \tag{0.5}$$

where P is the right real boundary point of $D_\rho(x_n, r)$. We claim that any choice of $(x_n)_{n\geq 1}$ with $x_n < x_{n+1}$ and

$$\frac{x_n + r}{1 + x_n r} \overset{(*)}{<} x_{n+1} < \frac{x_n + \frac{2r}{1+r^2}}{1 + x_n \frac{2r}{1+r^2}}$$

does the job.
i) To show (0.5), note that (*) implies that $x_n < x_{n+1}$ and

$$\begin{aligned}
\left| \frac{x_{n+1} - \frac{x_n + r}{1 + x_n r}}{1 - x_{n+1} \frac{x_n + r}{1 + x_n r}} \right| < r \quad &\overset{(*)}{\iff} \quad \frac{x_{n+1} + x_{n+1} x_n r - x_n - r}{1 + x_n r - x_{n+1} x_n - x_{n+1} r} < r \\
&\iff \quad x_{n+1}(1 + 2x_n r + r^2) < x_n + 2r + x_n r^2 \\
&\iff \quad x_{n+1} < \frac{x_n(1 + r^2) + 2r}{1 + r^2 + 2x_n r} \\
&\iff \quad x_{n+1} < \frac{x_n + \frac{2r}{1+r^2}}{1 + x_n \frac{2r}{1+r^2}}.
\end{aligned}$$

ii) It suffices to prove that $x_n \to 1$. Let $b \in]0,1]$ be the limit. Then, by (*),

$$b \leq \frac{b + r}{1 + br} \leq b.$$

Hence $b = 1$.
iii) Finally, $r < \rho(x_n, x_{n+1})$, since

$$\begin{aligned}
r < \rho(x_n, x_{n+1}) \quad &\iff \quad r < \frac{x_{n+1} - x_n}{1 - x_n x_{n+1}} \\
&\iff \quad r(1 - x_n x_{n+1}) < x_{n+1} - x_n \\
&\iff \quad \frac{r + x_n}{1 + r x_n} < x_{n+1}.
\end{aligned}$$

□