Proposition 4.76. *Let* $f \in H^\infty(\{|z| < R\})$ *and* $R > 1$. *Then*

$$-\frac{1}{\pi}\int_{1\le|w|<R}\frac{f(w)}{w-z}\,d\sigma_2(w) = \begin{cases}(1-R^2)f'(0) & \text{if } z=0\\ (1-R^2)\frac{f(z)-f(0)}{z} & \text{if } 0<|z|<1\\ (|z|^2-R^2)\frac{f(z)-f(0)}{z} + f(0)\frac{|z|^2-1}{z} & \text{if } 1\le|z|\le R\\ \frac{(R^2-1)f(0)}{z} & \text{if } |z|\ge R.\end{cases}$$

Proposition 4.82. *Let* $n, j \in \mathbb{N}$. *Then* [99]

$$I(z) := -\frac{1}{\pi}\int_{\mathbb{D}}\frac{w^n\,\overline{w}^j}{w-z}\,d\sigma_2(w) = \begin{cases}-\dfrac{z^{n-j-1}}{j+1}\left(1-|z|^{2(j+1)}\right) & \text{if } |z|<1 \text{ and } n-j-1\ge 0\\ \dfrac{z^n\,\overline{z}^{j+1}}{j+1} & \text{if } |z|<1 \text{ and } n-j-1<0\\ 0 & \text{if } |z|\ge 1 \text{ and } n-j-1\ge 0\\ \frac{1}{j+1}\frac{1}{z^{j+1-n}} & \text{if } |z|\ge 1 \text{ and } n-j-1<0.\end{cases}$$