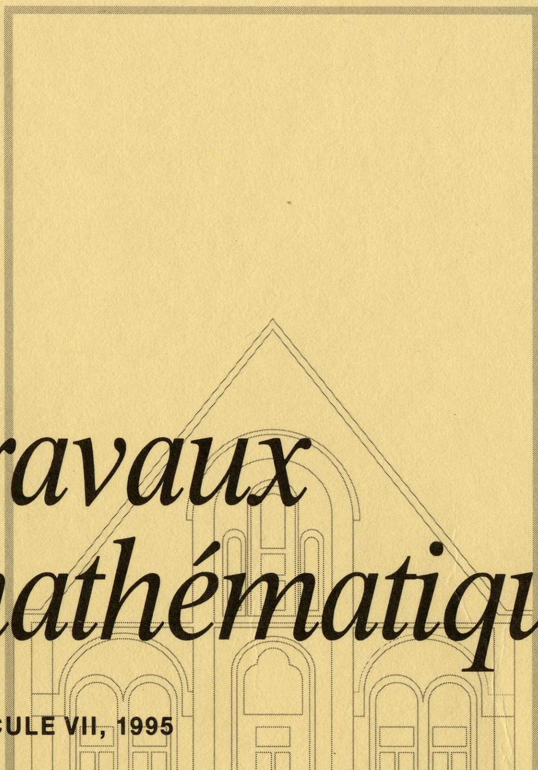

Travaux mathématiques

FASCICULE VII, 1995

Séminaire de mathématique de Luxembourg

Publications du CENTRE UNIVERSITAIRE LUXEMBOURG

Luxembourg 1995

Cyclic subspaces and eigenvectors of the hyperbolic composition operator

Raymond Mortini

Abstract

We give an explicit description of those singular inner functions which are eigenvectors of the hyperbolic composition operator C_ϕ on the Hardy space H^2. We also present sufficient conditions on H^2 functions f in order that the cyclic subspace K_f generated by f strictly contains the constants. One of these conditions involves the behavior of f on the maximal ideal space of H^∞. Results of Matache [7] are easily deduced.

AMS Classification number: 46 E 20, 47 B 38

Introduction

Let H^2 be the Hardy space of all those analytic functions f in the unit disk $\mathbb{D}$ for which

$$||f||_2^2 := \sup_{0<r<1} \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^2 \, d\theta$$

is finite and let H^∞ be the set of bounded analytic functions in $\mathbb{D}$ endowed with the supremum norm.

During the last 10 years, composition operators on H^2 of type $C_\phi f = f \circ \phi$ have been investigated quite extensively (see, e.g., the book [11] of Joel Shapiro). In view of their importance in connection with the invariant subspace problem (see § 1), we shall have in this paper a brief look on the structure of eigenvectors of C_ϕ for a hyperbolic automorphism ϕ of $\mathbb{D}$ as well as on the structure of the cyclic invariant subspaces K_f generated by some f in H^2. The reader, however, should be aware that most of the results of this paper are true for larger classes of selfmaps of the unit disk, although not explicitly mentioned.

Throughout this paper, ϕ will denote the hyperbolic automorphism

$$\phi(z) = \frac{z + \frac{1-r}{1+r}}{1 + \frac{1-r}{1+r} z}$$

with fixed points 1 and -1 and $\phi'(1) = r$ $(0 < r < 1)$. The point $z = 1$ is the attractive fixed point of ϕ.

The n-th iterate of ϕ will be denoted by ϕ_n, that is, $\phi_0(z) = z$, $\phi_{n+1} = \phi \circ \phi_n$ $(n \in \mathbb{Z})$. The inverse map of ϕ will be denoted also by ϕ^{-1} instead of ϕ_{-1}.

We recall that an *inner* function u is an analytic function in the ball of H^∞ for which the radial limits $\lim_{r\to 1} u(re^{i\theta})$ have modulus one almost everywhere. According to the canonical factorization theorems of Riesz and Smirnov (see [1], [4]), every inner function u can be decomposed as a product $u = e^{i\theta} B S_\mu$, where B is the Blaschke product associated with the zero sequence of u and where

$$S_\mu(z) = \exp\left[-\int \frac{e^{it} + z}{e^{it} - z}\, d\mu(t)\right]$$

is a singular inner function with positive finite Borel measure μ that is singular with respect to Lebesgue measure on the unit circle $T = \partial\mathbb{D}$.

§ 1 Sufficient conditions for the nonminimality of the cyclic subspaces K_f

Let C_ϕ be the hyperbolic composition operator defined by $C_\phi f = f \circ \phi$ for $f \in H^2$. It is well known that C_ϕ is a bounded linear operator (see [11]). For a function $f \in H^2$, let K_f be the cyclic subspace generated by f, that is, K_f is the closure of the linear subspace generated by $C_\phi^n f = f \circ \phi_n$ $(n = 0, 1, 2, \ldots)$. Obviously, every minimal invariant subspace of the composition operator C_ϕ has the form K_f for some $f \in H^2$. A remarkable theorem of Nordgren, Rosenthal and Wintrobe [9] says that the invariant subspace problem for Hilbert spaces is equivalent to the assertion that the minimal invariant subspaces of the hyperbolic composition operator C_ϕ are one-dimensional.

A positive answer to the following question Q1 would then give a positive solution to the invariant subspace problem.

Q1: Let K_f be a minimal invariant subspace for the hyperbolic composition operator C_ϕ. Does this imply that f is an eigenvalue?

Note that for every eigenvector f of C_ϕ we have $K_f = \langle f \rangle := \{\lambda f : \lambda \in \mathbb{C}\}$.

An obvious necessary condition for K_f, $f \neq$ const., being minimal is that K_f does not contain the constants. This follows from the fact that $\mathbb{C}$ is an invariant subspace for C_ϕ because $C_\phi 1 = 1$.

We present now some sufficient conditions on $u \in H^\infty$ which insure that $\mathbb{C} \subseteq K_u$.

Proposition 1.1. *Let $u \in H^\infty$. Assume that there exists a subsequence (ϕ_{n_k}) of (ϕ_n) such that $(u \circ \phi_{n_k})$ converges pointwise to a constant $c \neq 0$. Then $\mathbb{C} \subseteq K_u$.*

Proof. Since $\|u \circ \phi_n\|_2 \le \|u \circ \phi_n\|_\infty \le \|u\|_\infty$, the sequence $u \circ \phi_{n_k}$ converges pointwise, $\|\cdot\|_2$ boundedly to the constant function c. Because H^2 is a Hilbert space with a kernel function, this convergence is equivalent to the weak convergence in H^2. Hence $u \circ \phi_{n_k} \xrightarrow{w} c$.

By a general result in functional analysis (see [10], p. 67, Theorem 3.13), there exist convex combinations f_n of the functions $u \circ \phi_k$ for which f_n converges in the H^2-norm to c. Therefore $c \in K_u$. Since $c \neq 0$, we obtain that $\mathbb{C} \subseteq K_u$. ■

Remark. Proposition 1.1 extends Theorem 4 in [7]. In fact, the convergence there of the product $v = \prod_{n\ge 0} \lambda_n (u \circ \phi_n)$ for some $\lambda_n \in T$ implies that $\lambda_n(u \circ \phi_n(z)) \to 1$ as $n \to \infty$ for every z, for which the orbit $(\phi_n(z))_{n \ge n_0(z)}$ does not meet the zero set of u. Since $\{\lambda_n(u \circ \phi_n) : n \in \mathbb{N}\}$ is a normal family, Vitali's theorem and the countability of $\{\phi_n(z) : u(\phi_n(z)) = 0\}$ implies that $\lambda_n(u \circ \phi_n(z)) \to 1$ for every $z \in \mathbb{D}$. Take a sequence (λ_{n_k}) of (λ_n) for which $\lambda_{n_k} \to \lambda \in T$. Then $u \circ \phi_{n_k}(z) \to \bar{\lambda}$ for every $z \in \mathbb{D}$. Hence, by Proposition 1.1, $\mathbb{C} \subseteq K_u$.

Note also that the condition $v(0) \neq 0$ in [7] is not necessary. Moreover, in Proposition 1.1, u is not assumed to be an inner function.

Proposition 1.2. *Let $u \in H^\infty$. Assume that the radial limit $u^*(e^{i\theta}) = \lim_{r\to 1} u(re^{i\theta})$ exists at the attractive fixed point $e^{i\theta}$ of ϕ and that $u^*(e^{i\theta}) \neq 0$. Then $\mathbb{C} \subseteq K_u$.*

Proof. By ([11], p. 80) the orbit $(\phi_n(z))_{n\in\mathbb{N}}$ converges for every $z \in \mathbb{D}$ nontangentially to the attractive fixed point $e^{i\theta}$ of ϕ. By Fatou's theorem ([1],

p. 42), the existence of $u^*(e^{i\theta})$ implies that $u(z)$ converges nontangentially to $u^*(e^{i\theta})$ for $z \to e^{i\theta}$. Note that u is bounded. Hence $u(\phi_n(z)) \to u^*(e^{i\theta})$ as $n \to \infty$. By Proposition 1.1, $\mathbb{C} \subseteq K_u$. ∎

Remark. Proposition 1.2 extends Theorem 2 in [7], where it is assumed that u extends continuously at the attractive fixed point of ϕ, but is nonzero there.

Our next sufficient condition for the nonminimality of K_u ($u \in H^\infty$) involves the behaviour of u on the maximal ideal space (or spectrum) of H^∞, denoted by $M(H^\infty)$. We assume that the reader is familiar with Hoffman's theory as presented in the book of Garnett [1].

Recall that $\phi(z) = (z+a)/(1+az)$ denotes the hyperbolic automorphism with fixed points 1 and -1, $a = (1-r)/(1+r)$ $(0 < r < 1)$. Let $a_n = \phi_n(0)$, $n \in \mathbb{Z}$. Then $\phi_n(z) = (z+a_n)/(1+\bar{a}_n z)$. It is easy to prove that (a_n) is an interpolating sequence, that is (a_n) satisfies Carleson's condition

$$\inf_m \prod_{n:n\neq m} \left| \frac{a_n - a_m}{1 - \bar{a}_n a_m} \right| = \delta > 0$$

(see [9], p. 333). Moreover, $a_n = \dfrac{1-r^n}{1+r^n}$ $(n \in \mathbb{Z})$.

Let $m \in M(H^\infty)$ be a cluster point of $\{a_n : n \in \mathbb{N}\}$. By Hoffman ([3], [1], § 10), there exists a continuous bijective map L_m of $\mathbb{D}$ onto the Gleason part $P(m)$ of m so that $\hat{f} \circ L_m$ is analytic for every $f \in H^\infty$, where $\hat{f}$ denotes the Gelfand transform of f. Recall that $\hat{f}(m) := m(f)$, where $m \in M(H^\infty)$, and that $P(m) = \{x \in M(H^\infty) : \|x - m\| < 2\}$. Here $\|\cdot\|$ denotes the operator norm on the dual space $(H^\infty)^*$ of H^∞. Again by Hoffman ([3], [1], § 10),

$$L_m(z) = \lim_\alpha \frac{z + a_{n(\alpha)}}{1 + \bar{a}_{n(\alpha)} z}$$

(in the topology of $M(H^\infty)$), whenever $a_{n(\alpha)}$ is a subnet of $(a_n)_{n\in\mathbb{N}}$ converging to m.

We now prove the main result of this section.

Theorem 1.3. *Let $u \in H^\infty$, $a_n = \phi_n(0)$ and let $m \in M(H^\infty)$ be a cluster point of (a_n). Assume that $\hat{u} \circ L_m \equiv$ const. $\neq 0$. Then $\mathbb{C} \subseteq K_u$.*

Proof. Let $(a_{n(\alpha)})$ be a subnet of (a_n) converging to m. Then by Hoffman,

$$(u \circ \phi_{n(\alpha)})(z) \to \hat{u} \circ L_m(z) \equiv c$$

for every $z \in \mathbb{D}$. Thus the constant function c is a cluster point of the functions $u \circ \phi_n$ within the compact open topology on the space of holomorphic functions in $\mathbb{D}$. Because of the metrizability, there exists a subsequence (ϕ_{n_k}) so that $u \circ \phi_{n_k}(z) \to c$. By Proposition 1.1, $\mathbb{C} \subseteq K_u$. ∎

We remark that our previous results Propositions 1.1 and 1.2 are easily deduced from Theorem 1.3.

The proof also shows that if $u \in H^\infty$, then $u \circ L_m \in K_u$ for every $m \in \mathrm{cl}\,\{\phi_n(0) : n \in \mathbb{N}\}$. In particular, if $u \circ L_m$ is an eigenvector of C_ϕ, which is linearly independent of u, then K_u is not minimal.

§ 2 Eigenvectors of discrete singular inner type

The most simple singular inner function is the atomic inner function

$$S(z) = \exp\left(-\frac{1+z}{1-z}\right).$$

It plays an important role in the ideal theory of H^∞ ([4], [8]) and in the theory of composition of inner functions [2]. For example, if

$$S_\mu(z) = \exp\left(-\int \frac{e^{it}+z}{e^{it}-z}\, d\mu(t)\right)$$

is any singular inner function, then S is a left compositional factor of S_μ, that is, $S_\mu = S \circ v$, where v is an inner function (see [2], p. 256). the behaviour of iterates of S is studied in [5] and [6]. A particularly important class of singular inner functions is that for which the measure μ is discrete, that is, it has the form

$$\mu = \sum_{j=1}^{\infty} \varepsilon_j \delta_{\eta_j},$$

where δ_{η_j} is the Dirac measure at the point $\eta_j \in T$ and where $\sum_{j=1}^{\infty} \varepsilon_j < \infty$ $(\varepsilon_j \geq 0)$.

In this section we show that, for $\phi(z) = (z+a)/(1+az)$, $a = (1-r)/(1+r)$, $0 < r < 1$, no eigenvector of C_ϕ has S as a factor in the canonical factorization of H^2 functions. Moreover, no minimal invariant subspace of C_ϕ contains S.

On the other hand, there do exist plenty of eigenvectors of C_ϕ within the class of discrete singular inner functions; the associated Borel measures σ will have no mass at $\eta = \pm 1$. One example is the function $S \circ B^2$, where B is the Blaschke product with zeros $\phi_n(0)$, $n \in \mathbb{Z}$. We will give a complete description of that class of eigenvectors. We note that the corresponding problem for Blaschke products is trivial.

In fact, a Blaschke product B is an eigenvector of C_ϕ if and only if the zero set $E = Z(B)$ satisfies $\phi(E) = E$, counting multiplicities. In particular, if $a \in Z(B)$ has order $m \in \mathbb{N}$, then $Z(B)$ contains the whole orbit $\{\phi_n(a) : n \in \mathbb{Z}\}$, and each point in this orbit has the same multiplicity m.

More generally, the zero set of B has the form

$$\bigcup_a \bigcup_{n\in\mathbb{Z}} \{\phi_n(a)\},$$

where a runs through an at most countable discrete set of points in $\mathbb{D}$.

The following technical lemma is the main tool for the proof of our structural theorem on eigenvectors of C_ϕ of discrete singular inner type.

Lemma 2.1. *Let $S(z) = e^{-\frac{1+z}{1-z}}$ and let $T_{a,\theta} = e^{i\theta}\frac{a-z}{1-\bar{a}z}$ be a conformal automorphism of $\mathbb{D}$. Then*

$$S \circ T_{a,\theta}(z) = e^{-i\tilde{\alpha}} e^{-\alpha\frac{\beta+z}{\beta-z}},$$

where

$$\beta = T_{a,\theta}^{-1}(1) = \frac{ae^{i\theta} - 1}{e^{i\theta} - \bar{a}},$$

$$\alpha = \frac{1-|a|^2}{|1-e^{i\theta}a|^2} =: P(a, e^{-i\theta})$$

is the Poisson kernel and

$$\tilde{\alpha} = \frac{2\,\mathrm{Im}\,(e^{i\theta}a)}{|1-e^{i\theta}a|^2} =: Q(a, e^{-i\theta})$$

is the conjugate Poisson kernel. Note that

$$\gamma = \alpha + i\tilde{\alpha} = \frac{1+e^{i\theta}a}{1-e^{i\theta}a}.$$

Proof. We obviously have

$$\begin{aligned}\frac{1+e^{i\theta}\frac{a-z}{1-\bar{a}z}}{1-e^{i\theta}\frac{a-z}{1-\bar{a}z}} &= \frac{\frac{1+ae^{i\theta}}{\bar{a}-e^{i\theta}} - z\frac{\bar{a}+e^{i\theta}}{\bar{a}-e^{i\theta}}}{\frac{1-e^{i\theta}a}{\bar{a}-e^{i\theta}} - z} \\ &= \frac{z(\alpha - i\tilde{\alpha}) + \beta(\alpha + i\tilde{\alpha})}{\beta - z} \\ &= \alpha\frac{\beta+z}{\beta-z} + i\tilde{\alpha},\end{aligned}$$

which yields the assertion. ■

Proposition 2.2. *If for some $\alpha > 0$ the inner function S^α divides (within H^2) the function $f \in H^2$, then K_f is not minimal. In particular, no eigenvector of C_ϕ has S^α as a factor in the canonical inner-outer factorization.*

Proof. Let $f = S^\alpha g$ for some $g \in H^2$ and $\alpha > 0$. Without loss of generality, α may be chosen so that the inner factor of g has no point mass in 1.

By Lemma 2.1, we have $S \circ \phi_n = S^{1/r^n}$ for $n \in \mathbb{Z}$. Because $0 < r < 1$, the function S^α divides (within H^2) all the other functions $S^\alpha \circ \phi_n = S^{\alpha/r^n}$ for $n \in \mathbb{N}$. Since for any inner function u the subspace uH^2 is closed, we obtain that $K_f \subseteq S^\alpha H^2$.

Assuming K_f is minimal, we get that

$$S^\alpha g \circ \phi_{-n} = (S^\alpha \circ \phi_{-n})(g \circ \phi_{-n}) \in K_f \ \text{ for all } \ n \in \mathbb{N}.$$

Because the inner factor of $g \circ \phi_{-n}$ has no point mass in 1 (use Lemma 2.1 and note that $\phi_{-n}(1) = 1$), we can conclude that S^α divides $S^\alpha \circ \phi_{-n} = S^{\alpha r^n}$ for every $n \in \mathbb{N}$. This is absurd. Thus K_f is not minimal. ■

Remark. Similar results hold of course for the function $S_{-1}(z) := \exp\left(-\frac{1-z}{1+z}\right)$. Just interchange ϕ by ϕ^{-1}.

Theorem 2.3. *Let $\phi(z) = (z+a)/(1+az)$, $a = (1-r)/(1+r)$, $0 < r < 1$, be the hyperbolic automorphism with attracting fixed point $p = 1$ and $\phi'(1) = r$. Let*

$$P(a, e^{i\theta}) = \frac{1-|a|^2}{|e^{i\theta} - a|^2}$$

denote the Poisson kernel.

(1) *Let $\zeta_0 \in T \setminus \{-1, 1\}$, $\zeta_n := \phi_n(\zeta_0)$ $(n \in \mathbb{Z})$,*

$$\varepsilon_n = \left[\prod_{k=1}^{n} P(a, \zeta_k)\right]^{-1} \quad \text{for } n \in \mathbb{N},$$

$$\varepsilon_0 = 1,$$

$$\varepsilon_{-n} = \prod_{k=0}^{n-1} P(a, \zeta_{-k}) \quad \text{for } n \in \mathbb{N}.$$

Denote by orb $\zeta_0 = \{\phi_n(\zeta_0) : n \in \mathbb{Z}\}$ *the orbit of ζ_0. Put $\sigma := \sigma(\text{orb}\ \zeta_0) = \sum_{n=-\infty}^{\infty} \varepsilon_n \delta_{\zeta_n}$. Then S_σ is an eigenvector of C_ϕ.*

(2) *If the discrete singular inner function S_σ is an eigenvector of C_ϕ, then S_σ has the form*

$$S_\sigma = \prod_{k\in I} S^{\alpha_k}_{\sigma(\text{orb }\tau_k)},$$

where k runs through an at most countable set $I \subseteq \mathbb{N}$ so that each $\tau_k \in T\backslash\{-1,1\}$ belongs to a different orbit of ϕ and where the exponents $\alpha_k > 0$ satisfy $\sum_{k\in I} \alpha_k < \infty$.

Proof. (1) First we have to show that

$$\sum_{n=1}^{\infty} \varepsilon_n = \sum_{n=1}^{\infty} \frac{1}{\prod_{k=1}^{n} \frac{1-|a|^2}{|\zeta_k - a|^2}}$$

is convergent. In fact, as $\zeta_k \to 1$ for $k \to \infty$, we have

$$\frac{|\zeta_k - a|^2}{1-|a|^2} \le \frac{1-a}{1+a} + \varepsilon \quad \text{for} \quad k \ge n_0(\varepsilon).$$

If $\varepsilon > 0$ is chosen so that $(1-a)/(1+a) + \varepsilon < 1$, we see that

$$\left(\frac{1+a}{1-a}\right)^{n_0-1} \sum_{n=n_0}^{\infty} \left(\frac{1-a}{1+a} + \varepsilon\right)^{n-n_0}$$

is a convergent majorant of $\sum_{n=n_0}^{\infty} \varepsilon_n$. The convergence of

$$\sum_{n=1}^{\infty} \varepsilon_{-n} = \sum_{n=1}^{\infty} \prod_{k=0}^{n-1} \frac{1-|a|^2}{|\zeta_{-k} - a|^2}$$

is proved in the same way, noticing that $\zeta_{-k} \to -1$ as $k \to \infty$.

Thus

$$S_\sigma(z) = \prod_{n\in\mathbb{Z}} \exp\left(-\varepsilon_n \frac{\zeta_n + z}{\zeta_n - z}\right),$$

is well defined and it suffices to show that the functions

$$I_n \circ \phi = \exp\left(-\varepsilon_n \frac{\zeta_n + \phi}{\zeta_n - \phi}\right)$$

satisfy the relation

$$\begin{aligned} I_n \circ \phi(z) &= e^{i\omega_n} \exp\left(-\varepsilon_{n-1} \frac{\zeta_{n-1} + z}{\zeta_{n-1} - z}\right) \\ &= e^{i\omega_n} I_{n-1}(z) \qquad (n \in \mathbb{Z}) \end{aligned}$$

for some $\omega_n \in \mathbb{R}$. Fix $n \in \mathbb{Z}$. Then

$$I_n \circ \phi = \left[\exp\left(-\frac{1+\bar{\zeta}_n\phi}{1-\bar{\zeta}_n\phi}\right)\right]^{\varepsilon_n} = [S \circ \bar{\zeta}_n\phi]^{\varepsilon_n}$$

by taking suitable branches of the logarithm.

By Lemma 2.1,

$$S \circ \bar{\zeta}_n\phi(z) = e^{i\tilde{\alpha}_n} \exp\left(-P(a,\zeta_n)\frac{\zeta_{n-1}+z}{\zeta_{n-1}-z}\right).$$

Because $\varepsilon_n P(a,\zeta_n) = \varepsilon_{n-1}$, we obtain (1). Note that $(\omega_n)_n = (\varepsilon_n\tilde{\alpha}_n)_n$ is summable (use Lemma 2.1).

(2) This follows from (1), Proposition 2.2 and the fact that a discrete singular inner function S_σ is uniquely determined by the mass carried by the points of $\partial\mathbb{D}$. ∎

We conclude this note by giving some further remarks on Matache's paper [7]. In [7], Theorem 3, he shows that if u is inner and if v is a greatest common divisor of the family $\{u\circ\phi_n : n \in \mathbb{Z}\}$, then v is an eigenvector of C_ϕ whenever K_u is minimal. However, if $v = \gcd\{u\circ\phi_n : n \in \mathbb{Z}\}$, then v always is an eigenvector of C_ϕ. This has nothing to do with the hypothesis of K_u being minimal or not (see Proposition 2.4). The statement of Matache's theorem should have been: "If $v = \gcd\{u\circ\phi_n : n = 0,1,2\}$ and if K_u is minimal, then v is an eigenvector of C_ϕ".

The following proposition has been proved by my student U. Böttger when I was lecturing on the results of Matache.

Proposition 2.4. *Let $\phi \in \operatorname{Aut}\mathbb{D}$, u inner, and let v be a greatest common divisor of the family $\{u\circ\phi_n : n \in \mathbb{Z}\}$. Then v is an eigenvector of the composition operator C_ϕ.*

Proof. Since v divides $u\circ\phi_n$ for every $n \in \mathbb{Z}$, we obtain that $v\circ\phi$ divides $u\circ\phi_{n+1}$ for every $n \in \mathbb{Z}$. On the other hand, let w be a common divisor of $u\circ\phi_n$ for all $n \in \mathbb{Z}$. Then $w\circ\phi^{-1}$ divides $u\circ\phi_{n-1}$ for every $n \in \mathbb{Z}$. Hence $w\circ\phi^{-1}$ divides the greatest common divisor v of the family $\{u\circ\phi_j : j \in \mathbb{Z}\}$. Thus w divides $v\circ\phi$.

This reasoning shows that $v\circ\phi$ is a greatest common divisor of $\{u\circ\phi_n : n \in \mathbb{Z}\}$. Hence $v\circ\phi = e^{i\theta}v$ for some $\theta \in \mathbb{R}$. ∎

Remark. Matache's "should be" result (see above) is easily obtained from Proposition 2.4. In fact, if K_u is minimal, then K_u is the closed linear subspace generated by all of the functions $u \circ \phi_n$ $(n \in \mathbb{Z})$. Hence the gcd $\{u \circ \phi_n : n = 0, 1, 2, \ldots\} = \gcd \{u \circ \phi_n : n \in \mathbb{Z}\}$.

Acknowledgement. Part of this work was done while the author was visiting the University of Metz (France). He wants to thank Jean Ludwig and the university for their support.

References

[1] Garnett, J.B.: *Bounded Analytic Functions.* Academic Press, New York, 1981.

[2] Gorkin, P., Laroco, L., Mortini, R., and Rupp, R.: *Composition of inner funtions.* Results in Math. **25** (1994), 252–269.

[3] Hoffman, K.: *Bounded analytic functions and Gleason parts.* Ann. of Math. **86** (1967), 74–111.

[4] Hoffman, K.: *Banach Spaces of Analytic Functions.* Prentice Hall, Inc., Englewood Cliffs, N.J., 1962.

[5] Hong-Oh Kim: *Iteration of the unit singular inner function.* Bull. Korean Math. Soc. **25** (1988), 243–246.

[6] Hong-Oh Kim and Hoi Sub Kim: *Iteration, Denjoy-Wolff points and ergodic properties of point-mass singular inner functions.* J. Math. Anal. Appl. **175** (1993), 105–117.

[7] Matache, V.: *On the minimal invariant subspaces of the hyperbolic composition operator.* Proc. Amer. Math. Soc. **119** (1993), 837–841.

[8] Mortini, R.: *Closed and prime ideals of weak-Bezout type in H^∞.* J. Pure and Appl. Alg. **75** (1991), 63–73.

[9] Nordgren, E., Rosenthal, P., and Wintrobe, F.S.: *Invertible composition operators on H^p.* J. Funct. Anal. **73** (1987), 324–344.

[10] Rudin, W: *Functional Analysis.* 3rd ed., McGraw-Hill, 1991.

[11] Shapiro, J.: *Composition Operators and Classical Function Theory.* Springer Verlag, 1993.

Raymond Mortini
Mathematisches Institut I
Universität Karlsruhe
D-76128 Karlsruhe
Germany
AB05@DKAUNI2.Bitnet