UNIVERSAL SERIES AND FUNDAMENTAL SOLUTIONS OF THE LAPLACE EQUATION

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Abstract. Let ϕ be the standard fundamental solution of the Laplace operator on $\mathbf{R}^N, (N \geq 2)$. We prove the existence of universal series of the form

$$\sum_{k=0}^{\infty} c_k \phi(x - a_k) \tag{1}$$

$$\sum_{k=0}^{\infty} \sum_{|\alpha|=k} c_\alpha D^\alpha \phi(x - a) \tag{2}$$

in the space of functions that are harmonic in the neighborhood of a fixed compact set $K \subset \mathbf{R}^N$ with connected complement, or the space of functions that are harmonic on an open set $\Omega \subset \mathbf{R}^N$ that have an exhaustion by compact sets with connected complements. We also prove the existence of a serie of the form (**??**) which is convergent in $\mathbf{R}^N \setminus B(0, r)$ and is universally overconvergent in $B(a, r) \setminus \{a\}$. $a, a_k, k \in \mathbf{N}$ are fixed points lying outside the domain of definition of the harmonic functions. We also give some conditions for series in the form

$$\sum_{k=0}^{\infty} \sum_{|\alpha|=k} c_\alpha u_\alpha \tag{3}$$

to be universal in a metrizable topological linear space X.

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