# CLOSE UNIVERSAL APPROXIMANTS OF THE RIEMANN ZETA-FUNCTION

MARKUS NIESS

ABSTRACT. The Riemann zeta-function $\zeta(z)$ has the following well-known properties:

  (i) It is holomorphic in the complex plane except for a single pole at $z = 1$ with residue 1.

 (ii) The symmetry relation $\zeta(z) = \overline{\zeta(\bar{z})}$ holds for $z \neq 1$.

(iii) The functional equation
$$\zeta(z)\Gamma(z/2)\pi^{-z/2} = \zeta(1-z)\Gamma((1-z)/2)\pi^{-(1-z)/2}$$
holds.

(iv) It has a universal property due to Voronin (1975).

We show that arbitrarily close approximations of the Riemann zeta-function which satisfy (i)-(iii) may have a different universal property. Consequently, these approximations do not satisfy the Riemann hypothesis. Moreover, we investigate the set of all "'Birkhoff-universal"' functions satisfying (i)-(iii).