UNIVERSAL FUNCTIONS FOR COMPOSITION OPERATORS

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ABSTRACT. Let Ω be a planar domain and let $H(\Omega)$ be the Fréchet space of all holomorphic functions on Ω. Let X denote either $H(\Omega)$ or the unit ball $\mathcal{B} = \{f \in H(\Omega) : \sup_{z\in\Omega} |f(z)| \leq 1\}$ of $H^\infty(\Omega)$. A function $f \in X$ is said to be X-universal for a sequence, (ϕ_n), of selfmaps of Ω if $\{f \circ \phi_n : n \in \mathbb{N}\}$ is (locally uniformly) dense in X. Whereas the case of ϕ_n being an automorphism has been succesfully dealt with by many authors, we will study here the general case. It will be shown that for every domain $\Omega \subseteq \mathbb{C}$ for which $H^\infty(\Omega)$ is dense in $H(\Omega)$ there exists a sequence (ϕ_n) such that the family (C_{ϕ_n}) of composition operators admits $H(\Omega)$-universal functions. Moreover, if Ω is finitely connected, but not simply connected, then such a sequence of selfmaps cannot be eventually injective. On the other hand, if Ω is a domain of infinite connectivity, then a sequence of eventually injective selfmaps of Ω admits $H(\Omega)$-universal functions if and only for every Ω-convex compact subset K of Ω and every $n \in \mathbb{N}$ there is some $n \geq N$ such that $\phi_n(K)$ is Ω-convex and $\phi_n(K) \cap K = \emptyset$. The case of simply connected domains is considered, too. The problem of characterizing $\mathcal{B}$-universality for selfmappings of a non-simply connected domain is still open.