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Countably generated prime ideals in H^∞

The talk concerns joint work with Pamela Gorkin on the structure of prime ideals in the algebra H^∞ of bounded analytic functions in the open unit disk. In 1984 Gorkin and the speaker independently confirmed a conjecture of J. Kelleher and F. Forelli by showing that a nonzero prime ideal in H^∞ is finitely generated if and only if it is a maximal ideal of the form $M(z_0) = \{f \in H^\infty : f(z_0) = 0\}$ for some $z_0 \in \mathbb{D}$. These maximal ideals actually are principal ideals; they are generated by the single function $z - z_0$. In Mortini's thesis an example of a non-maximal, countably generated prime ideal is given: it is the ideal $I = I(S, S^{1/2}, S^{1/3}, \dots)$ generated by the n-th roots of the atomic inner function $S(z) = \exp\left(-\frac{1+z}{1-z}\right)$. This was the first explicit example of a non-maximal prime ideal in H^∞. Are there any other countably generated prime ideals in H^∞, apart from those given by inner rotations of the function S? For $|\sigma| = 1$, let $S_\sigma(z) = S(\overline{\sigma} z)$ be the atomic inner function with singularity at the point σ. In our latest joint paper (to appear in Math. Z.) we confirm the conjecture that a nonzero prime ideal I in H^∞ is countably generated if and only if either $I = M(z_0)$ for some $z_0 \in \mathbb{D}$ or if $I = I(S_\sigma, S_\sigma^{1/2}, S_\sigma^{1/3}, \dots)$ for some $\sigma \in \mathbb{C}$ with $|\sigma| = 1$. The proof uses maximal ideal space techniques and is based on some factorization theorems and on Suarez's result that H^∞ is a separating algebra.