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C-algebras generated by composition operators*

We discuss some recent work addressing the following general question: given a set of composition operators on a Hilbert function space H, what can be said about the C*-algebra they generate? We discuss the particular example of the C*-algebra $\mathcal{C}_\Gamma$, generated by the composition operators $\{C_\gamma : \gamma \in \Gamma\}$ acting on the Hardy space, where Γ is a discrete group of Möbius transformations. We prove that $\mathcal{C}_\Gamma$ contains the unilateral shift (and hence the compact operators $\mathcal{K}$), and there is an exact sequence

$$0 \to \mathcal{K} \to \mathcal{C}_\Gamma \to C(\partial\mathbb{D}) \times \Gamma \to 0$$

This result has many interesting consequences, for example by applying techniques from noncommutative geometry one can obtain index theorems for sums of weighted composition operators $\sum T_{f_\gamma} C_\gamma$.