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Boundary properties of uniform Frostman Blaschke products

A Blaschke product $B(z) = \prod_{n=1}^{\infty} \frac{|a_n|}{a_n} \frac{a_n - z}{1 - \overline{a_n} z}$ is uniform Frostman if the quantity

$$\sigma(B) = \sup_{\zeta \in \mathbb{T}} \sum_{n=1}^{\infty} \frac{1 - |a_n|^2}{|\zeta - a_n|}$$

is finite. Frostman showed that the sum is finite at $\zeta \in \mathbb{T}$ if and only if B and all of its subproducts have radial limits at ζ. The uniform Frostman condition $\sigma(B) < \infty$ imposes strong geometric constraints on the zero set of B. In particular, it is a finite union of interpolating sequences, and meets every Stolz region in a finite number of points, the number bounded by a constant depending only on $\sigma(B)$.

We discuss the boundary zero spectra of uniform Frostman Blaschke products and the limit properties of functions in the star-invariant subspaces K_B^p determined by these Blaschke products.